CFD in Turbomachinery - Harmonic Analysis Methods for Steady-State Applications

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There is a lot of talk about the application of harmonic-based methods to turbomachinery CFD analysis. Harmonic methods are used to accelerate CFD simulations of unsteady-state rotor-stator interaction and blade flutter.

In contrast to the mainstream unsteady-state applications this article illustrates a novel use of harmonic methods, namely to provide excellent *steady-state* rotor-stator interaction simulations. Sounds contradictory, but the seldom explored idea of harmonic analysis for the purpose of obtaining better steady-state turbomachinery simulations is a breath of fresh air after three decades of no choice beyond the industry stalwarts of mixing plane or frozen rotor analysis. The present article gives a brief overview of harmonic analysis in the context of turbomachinery CFD simulations, and explores the topic of harmonic steady-state rotor-stator interaction.

First a brief background: Turbomachinery involves rotating and stationary components, for example a turbocharger compressor comprising a radial compressor wheel with 10 – 20 rotor blades, spinning within a volute casing or an axial multistage turbine consisting of alternating rows of rotors and stator vanes. Regardless of the type of turbomachine, CFD simulations must account for the relative motion between the rotating blades and the stationary components, the so-called "rotor-stator interaction". The real fluid flow situation is always unsteady-state. Although CFD simulations can resolve these unsteady-state flow fields, it is often at too high a cost. The alternative are rotor-stator interaction models that approximate the flow as steady-state. Analysts must choose the right balance between computational time and the fidelity or accuracy of the simulation. Transient rotor-stator interaction simulations are often 100 to 1,000 times more expensive than their steady-state rotor-stator counterparts, i.e. computations are lasting days, rather than an hour.

A lot of research has been focused in the past decade on accelerating transient rotor-stator simulations using specific methods to address non-integer blade count, the "pitch change" problem, between blade rows. Non-integer blade count is structurally required to control blade vibrations but computationally confounding as it may force the simulation to include full wheels with all blades.

Harmonic analysis takes the computational advantage provided by time-transient pitch change methods one step further. It solves only for the final quasi-periodic flow state but not its evolution. This is done at a small number of points in time or time planes, distributed uniformly within the rotor blade passing period, as shown in Figure 1. The time derivative on a given time plane is computed by a high-order curve fit in time, passing a discrete Fourier series through all of the solved-for time planes. More time planes allow for higher-order Fourier series which improve accuracy. As the number of time planes is increased, the harmonic analysis will approach a well converged time-transient simulation of sufficiently small time step.

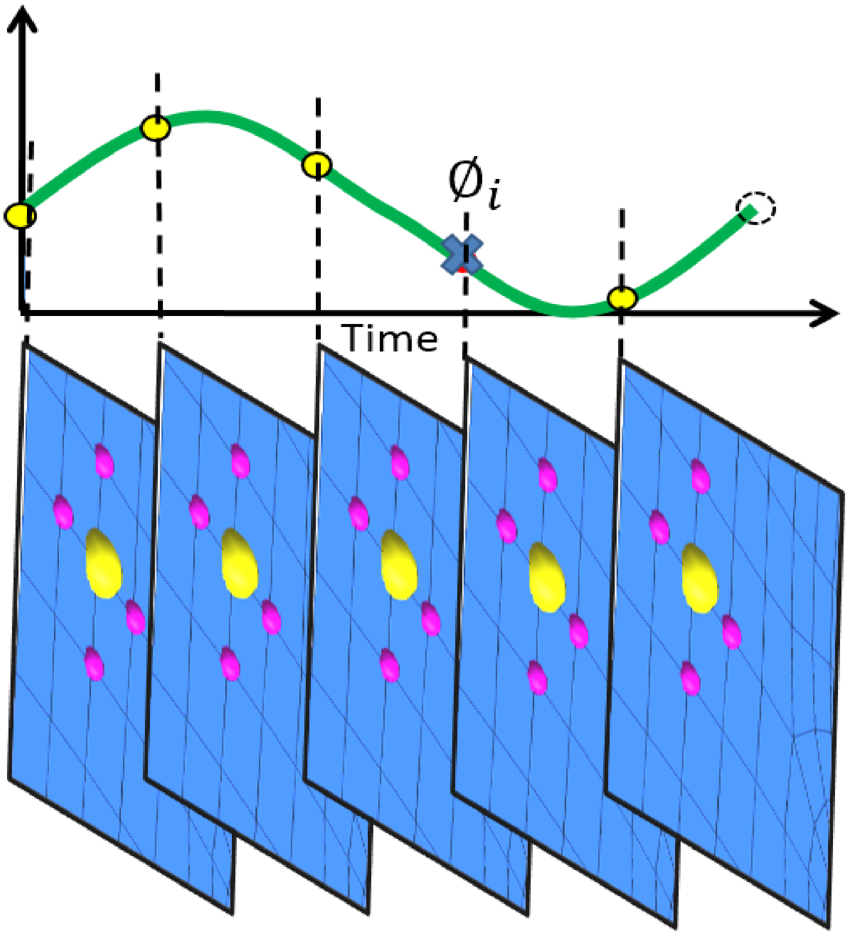

Figure 1: Time planes in harmonic analysis

With this basis in mind, how might harmonic analysis be applied to steady-state rotor-stator interaction? Well, every harmonic analysis provides a Fourier series approximation of the transient signal in the form

$$\theta(t) = A_0 + \sum_{m=1}^{M} [A_m \cos(m\omega t) + B_m \sin(m\omega t)]$$

where the transient variation of a scalar, $\theta(t)$, is expressed in terms of its mean value, A_0, plus its M harmonics about a base frequency, ω, the rotor-stator blade passing period. Suppose an analyst is only interested in the mean flow, which is a natural outcome of a harmonic analysis in form on the leading A_0 term, and is not caring how accurately the temporal variation is resolved or predicted. Then the question becomes, how few time planes, and hence how few harmonics, are necessary to provide a good approximation of the mean flow?

ISimQ has explored two cases to test the idea of using harmonic analysis to model steady-state rotor-stator interaction. The first is a transonic process compressor designed by PCA Engineers[1], see Figure 2. Its high-pressure ratio compressor is challenging for simulation methods, as there are distinct unsteady-state interactions between the 18 rotor blades and the 17 close-coupled diffuser vanes.

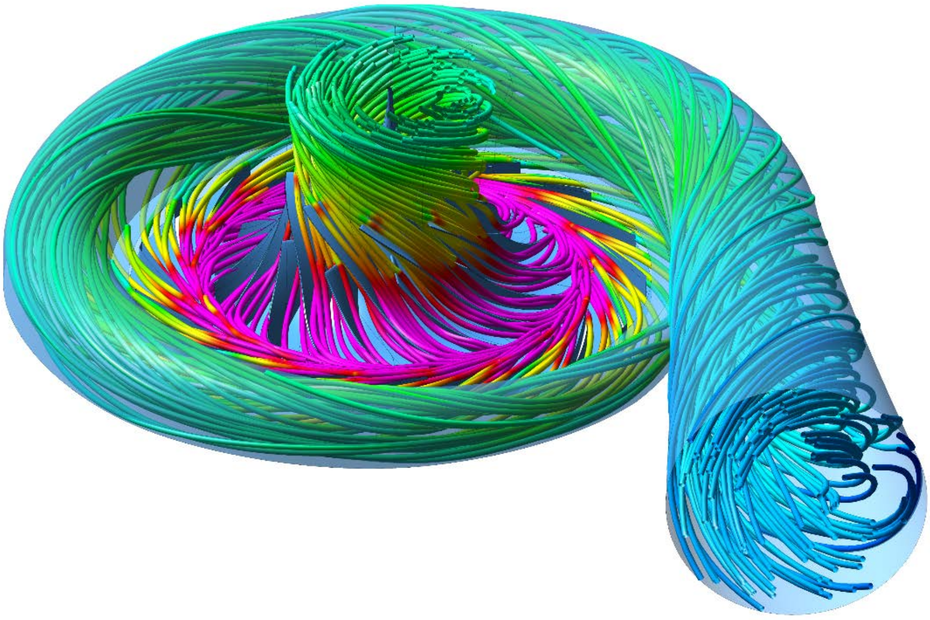

Figure 2: Radial compressor from PCA Engineers

Figure 3 compares simulations with just three harmonics (7 time planes) to a full unsteady-state simulation. It shows an instantaneous mid-span total pressure field from the unsteady-state simulation, and the central time plane from the harmonic solution. The overall character of the flow is captured quite well by the harmonic solution. But now look at the mean flow predictions in Figure 4, comparing the time average of the unsteady-state solution to the A_0 coefficient of the harmonic analysis. The time averages of the two predictions are very close!

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[1] http://www.pcaeng.co.uk/

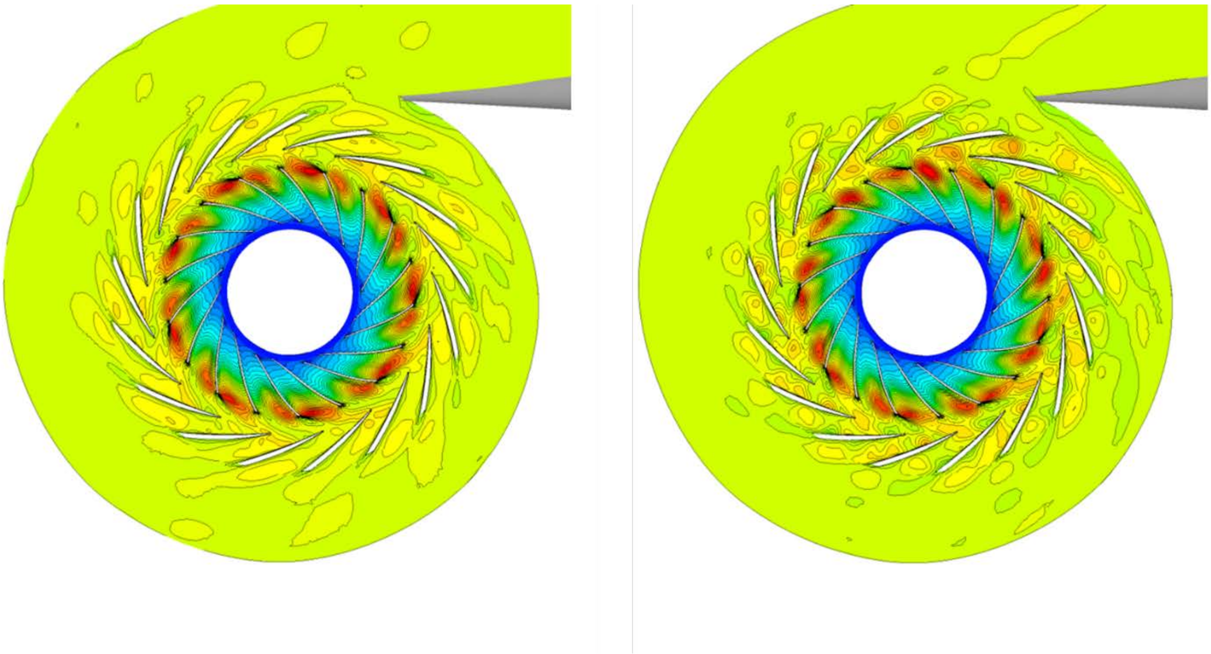

Figure 3: Unsteady-state compressor flow simulations; left side: unsteady-state snapshot; right side: harmonic $n = 3$ time planes

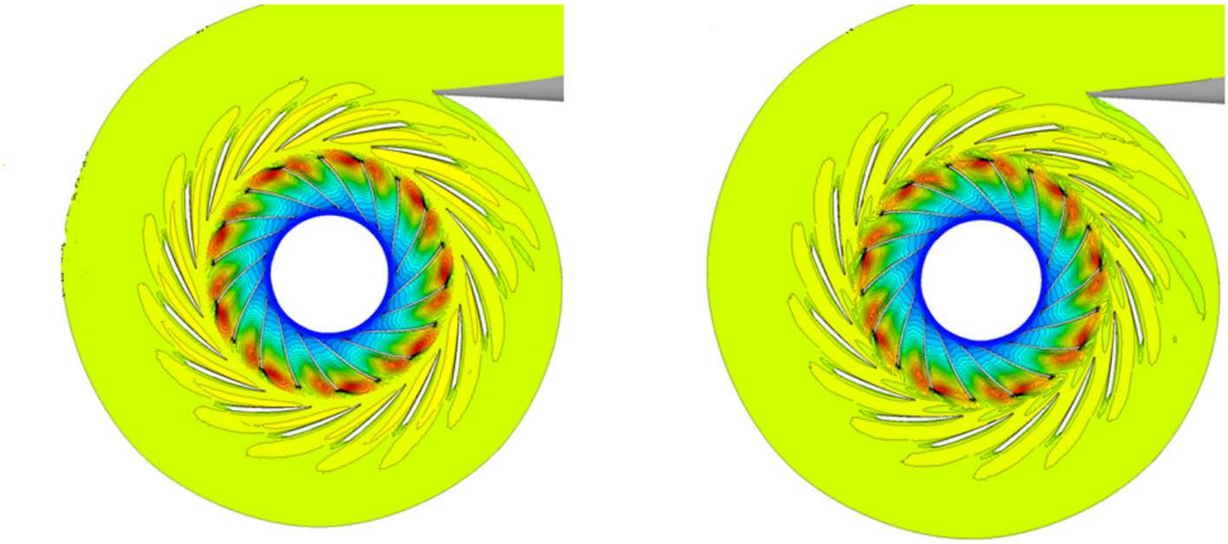

Figure 4: Steady-state compressor flow simulations; left side: averaged unsteady-state result; right side: harmonic $n = 3$ time planes, A_0

At the compressor's best efficiency point, the predicted mass flow rate between the unsteady-state and the harmonic analysis simulations is within 0.15 %, the pressure ratio within 0.02 % and the efficiency within 0.12 points. However, the unsteady-state simulation required 11 times more computational effort than the harmonic analysis.

The key point is that the harmonic analysis predicts the time-averaged flow with no need to introduce a "steady-state" model for the rotor-stator interface. There is no mixing plane involved, nor is a frozen rotor interface required. Because the analysis does not need a steady-state interface model, the harmonic mean flow analysis is also possible where traditional mixing plane or frozen rotor interfaces fail, for example when there is local reverse flow at the mixing plane interface or where strong flow features such as shocks and wakes cross the interface.

In a second example, the approach was validated on Notre Dame Turbomachinery Laboratory's[2] NDTAC transonic axial compressor stage consisting of 20 rotor blades and 43 stator values. This highly loaded stage, with a pressure ratio of approximately 1.5 and tip Mach number of 1.3, features complex transient blade row interactions, making for a challenging test of harmonic analysis. The case was solved using ANSYS CFX's profile transformation (PT) method using both unsteady-state and harmonic analysis with $M = 3$ harmonics.

The mean performance of the stage was closely predicted by both approaches, as seen in Figure 5, even for challenging operating points just prior to compressor stall. Figure 6 compares mid-span total temperature snapshots of the harmonic and unsteady-state simulations. The harmonic analysis resolves flow features similar to those seen in the time-transient simulation, and all this from only three harmonics, i.e. 7 time planes, as compared to the many thousands of time steps required by the time-transient simulation. In this case the harmonic analysis was seven times faster than the unsteady-state simulation.

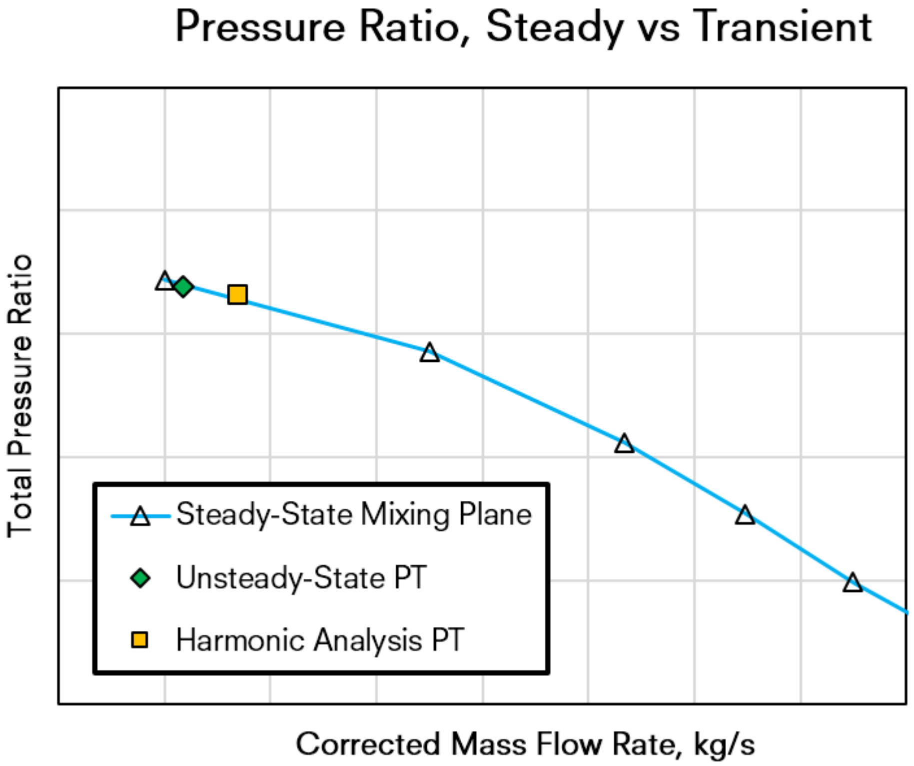

Figure 5: Total pressure ratio just before stall by steady-state, unsteady state and harmonic analysis

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[2] http://turbo.nd.edu/

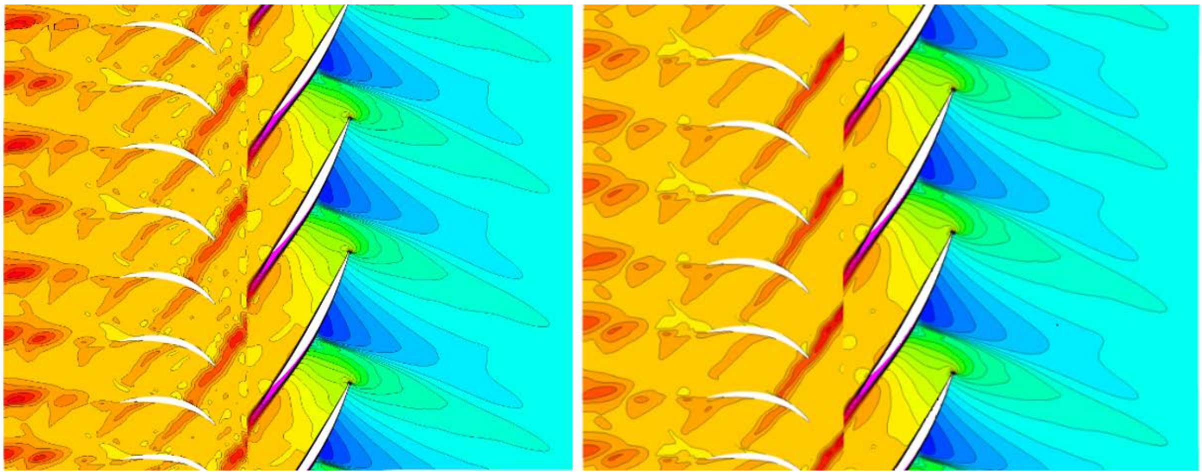

Figure 6: Transonic compressor simulations; left: harmonic $n = 3$; right: unsteady-state snapshot

After several decades of having no new options for steady-state rotor-stator interaction, the ability to use low-resolution harmonic analysis is an exciting development. Maybe someday in the near future mixing plane models for turbomachinery analysis finally will get some well-deserved rest.